Abstract

The basic formulation for finite element modeling of metal forming processes is briefly recalled with the aim of treating the case of multi-body interactions. This situation occurs when the tools are considered as deformable, when the work-piece includes several materials or when the physical structure is analyzed at the micro scale. The classical approach utilizes separate meshes for each body and the contact is enforced using different numerical methods: the complete coupling, the master and slave approach and a quasi-symmetrical formulation. The single mesh method with different constitutive equations corresponding to each material is more computationally effective, but its use is restricted to the cases when the contact between the different bodies does not evolve. Finally the Euler formulation can be used with a level set method for the description of the interfaces between different materials and its application to recrystallization for example.

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1. Introduction

Simulation of metal forming processes started in the 1970’s, mainly in academic laboratories for 2D work-pieces treating: hydrostatic extrusion by Iwata et al, the analysis of relative slip on the tools by Lee and Kobayashi, and large deformations of viscoplastic materials by Zienkiewicz and Godbole. In the 1980’s the use of simulation codes started for industrial forming applications, while 3D forging developments started in laboratories by Surdon.
and Chenot. Now commercial finite element computer codes are developed and maintained in several software companies which favor their diffusion in large and medium enterprises. Simulation is widely recognized by the engineers as a powerful tool for effective design and optimization of forming processes. More complex and realistic processes can be treated, including coupling of the work-piece with deformable tools, forming of a part composed of different materials and computation at the micro scale to predict evolution of the metallurgical structure. The main purpose of the paper is to review and analyze different approaches for treating the numerical problems related to plastic deformation of several materials. This problem was addressed numerically for structural computation by Habraken and Cescotto, with a symmetric formulation between separate meshes of the bodies in contact, and for metal forming by Fourment et al. The symmetric method is appropriate for structural computation with small displacements, but it may introduce too many constraints and results in a too stiff interface when large relative displacements are involved. A widely spread approach is the “master and slave” approximation introduced by Hallquist et al., in which the contact conditions are imposed only on one of the discretized boundaries. This method allows satisfactory computation of the boundary but, for avoiding unrealistic mesh penetration, the mesh of the “slave” interface must be more refined than that of the “master”. This problem was treated by Fourment who developed a quasi-symmetrical approach. Another method presented by Beraudo was shown to decrease computer time by simply introducing meshes with coincident nodes at the interface between materials. Finally the Euler approach, with a refined description by level sets of interfaces between materials, was used by Resk et al. to simulate the evolution of physical entities at the micro scale.

2. Mechanical and numerical formulations

For a more complete introduction to numerical simulation of metal forming see Wagoner and Chenot.

2.1. Mechanical formulation

For an elastoplastic or elastic viscoplastic deformation, we use an additive strain rate decomposition of the form:

\[ \dot{e} = \dot{e}^e + \dot{e}^p. \]  

where \( \dot{e}^e \) is the elastic strain rate and \( \dot{e}^p \) the plastic or visco plastic strain rate; \( \lambda \) and \( \mu \) the Lamé coefficients. The elastic law is written with the Jauman derivative for objectivity (denoted by \( d_J \)):

\[ \frac{d_J \sigma}{dt} = \lambda \text{tr}(\dot{e}^e) + 2\mu \dot{e}^e. \]  

A viscoplastic law is expressed by a power law of the form:

\[ \dot{e}^p = 1 / K \left( (\sigma - R) / K \right)^{1/m - 1} \sigma. \]  

where \( \sigma \) is the deviatoric stress tensor, \( \sigma \) the usual equivalent stress, K the consistency and \( m \) the strain rate sensitivity.

For an incompressible viscoplastic flow, with small elastic strains, it is desirable to utilize a mixed formulation in the domain \( \Omega \) of the part. For any virtual velocity field \( v^* \) it is written:

\[ \int_\Omega \sigma^*:\dot{e}^*dV - \int_{\partial \Omega} \text{div}(v^*)dV - \int_{\partial \Omega} \tau v^*dS = 0. \]  

For any virtual pressure field \( p^* \), the mass conservation constraint is enforced by:
Equations (4) and (5) are often rewritten in term of displacement and stress increments.

Now we consider two different bodies contact and a material point M on their interface $\Gamma$. In order to allow relative slip or separation of the bodies, we define the material velocities $v_a$ and $v_b$ on both sides of the interface with normal vector $n$ to $\Gamma$ pointing toward body denoted “b”. The rate form of the non-penetration condition is expressed for the continuous problem by the inequality.

$$ (v_a - v_b) n = \Delta v n \leq 0. $$

(6)

The normal stress $\sigma_n = (\sigma n)n$ is compressive or null, i.e. $\sigma_n \leq 0$, when the two bodies can be in contact, or separate without adhesion. For a sliding contact at the interface of the two bodies, a friction law is defined, which can be modelled by a generalized Coulomb law, for example we put:

$$ \tau = \mu(\sigma n) \Delta v / |\Delta v|^{\mu}. $$

(7)

### 2.2. Finite element discretization

The pressure field is discretized using linear tetrahedral elements, and a bubble function is added to the velocity or the displacement field, in order to stabilize the solution. We obtain a set of non-linear equations that can be solved globally using a Newton-Raphson algorithm on the set of nodal unknowns: velocity $V$ and pressure $P$, or increments of displacement $\Delta U$ and of pressure $\Delta P$.

### 3 Multi body problems with separate meshes

#### 3.1. General formulation

Consider now two bodies with discretized domains $\Omega_a$ and $\Omega_b$ and their contact between their interfaces $\partial \Omega_a$ and $\partial \Omega_b$. For any material point M located on $\partial \Omega_a$, with normal $n_a$ pointing toward the exterior of $\Omega_a$, we define the distance $h_{ab}$ to $\partial \Omega_b$ along $n_a$. It is negative if M is outside of $\Omega_b$ and positive if it is in the interior, then non-penetration is expressed by: $h_{ab} \leq 0$. We define in the same way the distance $h_{ba}$ along the normal vector $n_b$ from $\partial \Omega_b$ of any point to $\partial \Omega_a$. The contact constraint can be treated numerically utilizing the Lagrange multiplier method, or using the penalty method which may allow only a very small penetration. The basic idea is to introduce a penalty factor $\rho > 0$ and define a normal contact force by:

$$ f_c = -\rho h' n, \text{ with } h' = \frac{1}{2}(h + |h|). $$

(8)

It is taken into account by adding the following equations to Eq. (4); for any virtual velocity fields $v_a^*$ and $v_b^*$:

$$ -\rho \int_{\partial \Omega_a} h'_{ab} n_a v_a^* dS - \rho \int_{\partial \Omega_b} h'_{ba} n_b v_b^* dS. $$

(9)

If a relative slip of the two bodies will occur, a friction stress similar to Eq. (7) is added. This formulation is called symmetrical as the role of the two bodies is similar. It is easy to convince oneself that it is too constraining as the number of constraints imposed by Eq. (9) is equal to the number of degrees of freedom of both interfaces.
Consequently the interfaces are too rigid and this formulation leads to a kind of locking, which is acceptable only when displacements are very small.

3.2. Master and slave approximation

Only one integral is introduced to enforce the contact condition, i.e. the “slave” surface $\partial \Omega_a$ must not penetrate the “Master” surface $\partial \Omega_b$. For any virtual velocity field $v_a^*$:

$$-\rho \int_{\partial \Omega_a} h_{ab} n_a v_a^* dS. \quad (10)$$

Which leads to a satisfactory solution provided that the mesh of $\partial \Omega_a$ is more refined than that of $\partial \Omega_b$ and a reduced integration method is introduced to avoid over constrained contact condition. Another popular method for usual forming problems is to impose contact by nodal conditions i.e. to replace Eq. (10) by a sum of nodal penalty forces. It is applied to the coupling between work-piece and elastic tools and to forming of a part composed of different materials which can exhibit a relative slip.

3.3. Quasi symmetrical method

The objective of this approximation is to avoid the introduction of too many constraints and to design a formulation as close as possible to the symmetric one. For that purpose the second integral in Eq. (9) is replaced by an approximation using the projection $\pi_{ab}(v_a^*)$ on the space of velocities $v_b$. For any virtual velocity field $v_a^*$, Eq. (9) is replaced by the following additional terms:

$$-\rho \int_{\partial \Omega_a} h_{ab} n_a v_a^* dS - \rho \int_{\partial \Omega_b} h_{ba} n_b \pi_{ab}(v_a^*) dS. \quad (11)$$

This approach was first proposed by Fourment with a slightly different formulation.

4. Single mesh approach

The single mesh method was applied to extrusion coating. Numerical simulation allows us to predict heterogeneity of deformation along the axis of the part, with an expansion which can be seen at the left in Fig. 1, followed by a small necking.

![Fig. 1. Extrusion coating of two materials (TiAl in blue, steel in red).](image)

The computational advantage of the method is illustrated in Fig. 2 by comparison of the CPU time between the single mesh method and the multi body approach for a case of bending of a tube composed of 4 different materials. We can observe that the saving in term of CPU time is about 75%, which is mainly due to less computation when the contact algorithm is not activated and the resolution method is converging faster.
5. Eulerian formulation

In the Eulerian method, not only a single mesh is used, but it is stationary and does not follow the material deformation as in the usual updated Lagrangian formulations. The interface between different bodies is defined by a moving surface $\Gamma$, described by a level set function, interpolated on the fixed mesh. The interface $\Gamma$ is then given by the level 0 of the level-set function $\phi$, which is usually a signed distance function:

$$\phi(x) = d(x, \Gamma), \ x \in \Omega \text{ and } \Gamma = \{ x \in \Omega, \ \phi(x) = 0 \}. \quad (12)$$

Fig. 3 shows the case where a polycrystal made of 1000 grains is described thanks to a level-set function for each grain. The considered FE mesh is not conformed to the grain interfaces but anisotropically refined at the grain interfaces. Thus, each domain (grain) is described implicitly. The evolution of the interfaces of the grains is simply computed by a time integration scheme with the local velocity field, which can be for example a mechanical velocity in context of crystal plasticity finite element method (CPFEM), or a grain boundary migration velocity field due to recrystallization or grain growth phenomena. In order to preserve the accuracy of the definition of $\Gamma$, an adaptive remeshing is performed in the vicinity of the interface. For predicting the local microstructure evolution during a forming process, the REV is subjected to a thermal and mechanical history that will be modelled with classical physical laws. Fig. 4 describes the case where a REV is deformed in context of CPFEM. As illustrated the local deformation can be very heterogeneous due to texture of the considered material.
Fig. 4. Deformation of a REV composed of grains. Left: strain localization. Right: stored energy.

Using the strain and stored energy distributions, static recrystallization is simulated by introducing germs that will initiate a new microstructure. This is illustrated in Fig. 5 in a 2D section for a set of grains with different orientations.

Fig. 5. 2D section. Left: stored energy. Right: potential sites of nucleation.

6. Conclusions

Several methods for analyzing the multi body contact problem were briefly presented, including the classical multi mesh approach, the single mesh technique and the Euler formulation with a level set description of the interfaces. Each of these methods has shown specific advantages, depending on the applications.

References